Exercise Sheet #8

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P1. This exercise shows that the locally finite assumption cannot be removed in Theorem 6.15. (The measure μ in part **(b)** is not locally finite). Let $(X,\mathcal{U})$ be the topological space defined by $X := \mathbb{N} \cup \{\infty\}$ and

$$\mathcal{U} := \{U \subset X \mid U \subset \mathbb{N} \text{ or } |U^c| < \infty\}.$$

Thus $(X,\mathcal{U})$ is the (Alexandrov) one-point compactification of the set $\mathbb{N}$ of natural numbers with the discrete topology. (If $\infty \in U$ then the condition $|U^c| < \infty$ is equivalent to the assertion that U^c is compact).

(a) Prove that $(X,\mathcal{U})$ is a compact Hausdorff space and that every subset of X is σ-compact. Prove that the Borel σ-algebra of X is $\mathcal{B} = 2^X$.

(b) Let $\mu : 2^X \to [0,\infty]$ be the counting measure. Prove that μ is inner regular, but not outer regular.